



Electromagnetic mass model admitting conformal motion*

Saibal Ray^{a*}, A A Usmani^b, F Rahaman^c, M Kalam^d and K Chakraborty^e

^aDepartment of Physics, Barasat Government College, Barasat-700 124,
North 24 Parganas, India

^bDepartment of Physics, Aligarh Muslim University, Aligarh-202 002, Uttar Pradesh, India

^cDepartment of Mathematics, Jadavpur University, Kolkata-700 032, India

^dDepartment of Physics, Netaji Nagar College for Women, Regent Estate,
Kolkata-700 092, India

^eDepartment of Mathematics, Jadavpur University, Kolkata-700 032, India

E-mail saibal@iucaa.ernet.in

Received 22 June, 2008, accepted 13 August, 2008

Abstract We study charged fluid spheres under the 4-dimensional Einstein-Maxwell space-time. The solutions thus obtained admitting conformal motion. We also investigate whether the solutions set provide electromagnetic mass models such that the physical parameters including the gravitational mass arise from the electromagnetic field alone. In this connection three cases are studied here in detail with the propositions (1) $\rho = -p$, (2) $\sigma e'^{1/2} = \sigma_0$ and (3) $8\pi p - E^2 = p_0$ where ρ, p, σ are respectively the usual matter density, fluid pressure and charge density of the spherical distribution. Based on these assumptions several features are explored which seems physically very interesting.

Keywords Charged fluid sphere, electromagnetic mass, conformal motion

PACS Nos. 04-20 -q, 04-20 Jb, 98-80 Hw

1. Introduction

There is a fairly long history of investigations regarding the nature of the mass of electron. While studying the interaction of charged particles Thomson [1] found that the kinetic energy of a charged sphere increases by its motion through a medium of finite specific inductive capacity. He pointed out that the increase in the kinetic energy was due to the self induced magnetic field of the charged sphere and came to the conclusion that "the effect of electrification is the same as if the mass of the sphere were increased ...". This

*Corresponding Author

[†]Preprint submitted to Elsevier Preprint

investigation was improved upon by Heaviside [2] showing that the mass of a uniformly moving charged body varied with velocity. Searle [3] extended this again as that the energy of a charged body and hence its mass increases with velocity. Later on Kaufmann [4–8] conducted a series of experiments on beta rays and established the dependence of the electron mass on velocity. He [5] showed that one-third of the fast moving electron mass is of electromagnetic origin. However, Abraham [9] speculated that the electron mass was completely of electromagnetic origin. This result insisted Kaufmann [8] to re-analyze his experimental data and he found that the mass of the electron is purely of electromagnetic origin.

Under this illuminated background, therefore, Lorentz [10] proposed his model for extended electron and conjectured that "there is no other, no 'true' or 'material' mass", and thus provides only 'electromagnetic masses of the electron'¹. Wheeler [12] and Wilczek [13] pointed out that electron has a "mass without mass" whereas Feynman, Leighton and Sands [14] termed this type of models as "electromagnetic mass models". Following the idea of *electromagnetic mass* (EMM) a lot of works have been carried out by several authors [15–23] under the framework of general relativity where space-time geometry is assumed to be associated with the presence of matter.

On the other hand to search the natural relation between geometry and matter through the Einstein equations it is convenient to use inheritance symmetry. The well known inheritance symmetry is the symmetry under conformal killing vectors (CKV) which can be given by $L_{\xi}g_{ij} = \psi g_{ij}$ where L is the Lie derivative operator and ψ is the conformal factor. Here it is supposed that the vector ξ generates the conformal symmetry and then the metric g is conformally mapped onto itself along ξ . In this connection it is to be noted that neither ξ nor ψ need to be static even though one considers a static metric [24,25]. It can be seen in the literature that due to this and several other properties CKVs provide a deeper insight into the space-time geometry connected to astrophysical and cosmological realm [26–31].

Now the above works on EMM models [15–23] have been performed either in 4D or higher dimensional space-time under general relativity. Therefore, our motivation in the present investigation is to include CKV and see whether conformal motion admits EMM models or not. To do so we have considered here static spherically symmetric charged perfect fluid distribution under 4D general relativity and studied three cases : (1) $\rho = -\rho$, (2) $\sigma e^{\lambda/2} = \sigma_0$ and (3) $8\pi\rho - E^2 = \rho_0$ where ρ, p, σ are respectively the usual matter density, fluid pressure and charge density of the spherical distribution. The scheme of the present investigations are as follows : Sections 2 and 3 are related to the Einstein field equations with CKV and their solutions for the three cases. In the concluding Section 4, some remarks are made.

¹ It would be historically very interesting to note that even after these profound theoretical and experimental results Einstein [11] himself believed that "... of the energy constituting matter three-quarters is to be ascribed to the electromagnetic field, and one-quarter to the gravitational field".

2. Einstein-Maxwell field equations in 4-dimensional space-time

The static spherically symmetric space-time is taken as

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (1)$$

where ν and λ are the metric potentials and functions of radial coordinate r only.

Now, the Einstein field equations for the case of charged fluid source are

$$G'_j = R'_j - \frac{1}{2} g'_j R = -\kappa [T_j^{(m)} + T_j^{(em)}] \quad (2)$$

where $T_j^{(m)}$ and $T_j^{(em)}$ are, respectively, the energy-momentum tensor components for the matter source and electromagnetic field. The explicit forms of these tensors are given by

$$T_j^{(m)} = (\rho + p) u^j u_j + p g'_j, \quad (3)$$

$$T_j^{(em)} = -\frac{1}{4\pi} \left[F_{jk} F^{jk} - \frac{1}{4} g'_j F_{kl} F^{kl} \right] \quad (4)$$

where ρ, p and u^j are, respectively, matter-energy density, fluid pressure and velocity four-vector of a fluid element (with $u_j u^j = 1$).

The Maxwell electromagnetic field equations are given by

$$[(-g)^{1/2} F^{ij}]_{,j} = 4\pi J^i (-g)^{1/2}, \quad (5)$$

$$F_{[ij,k]} = 0, \quad (6)$$

where the electromagnetic field tensor F_{ij} is related to the electromagnetic potentials through the relation $F_{ij} = A_{i,j} - A_{j,i}$, which is equivalent to the eq. (6). In the above equation J^i is the current four-vector satisfying $J^i = \sigma u^i$, where σ is the charge density, and $\kappa = 8\pi$. We have considered the relativistic unit for which $G = c = 1$. Here and in what follows a comma denotes the partial derivative with respect to the coordinates.

In view of above, therefore, the Einstein-Maxwell field equations can be given as

$$e^{-\lambda} \left[\frac{\lambda'}{r} - \frac{1}{r^2} \right] + \frac{1}{r^2} = 8\pi\rho + E^2, \quad (7)$$

$$e^{-\lambda} \left[\frac{1}{r^2} + \frac{\nu'}{r} \right] - \frac{1}{r^2} = 8\pi p - E^2, \quad (8)$$

$$\frac{1}{2}e^{-\lambda} \left[\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda') \right] = 8\pi\rho + E^2 \quad (9)$$

and

$$(r^2 E)' = 4\pi r^2 \sigma e^{\lambda/2}. \quad (10)$$

The electric field E in the above eq. (10) can equivalently be expressed in the form

$$E(r) = \frac{1}{r^2} \int_0^r 4\pi r^2 \sigma e^{\lambda/2} dr = \frac{q(r)}{r} \quad (11)$$

where $q(r)$ is the total charge of the sphere under consideration.

It is interesting to note that the eqs. (7) and (8) provide an essential relationship between the metric potentials and the physical parameters ρ and p as follows

$$e^{-\lambda} (\nu' + \lambda') = 8\pi r(\rho + p). \quad (12)$$

Again, eq. (7) may be expressed in the general form as

$$e^{-\lambda} = 1 - \frac{2M(r)}{r}, \quad (13)$$

where $M(r)$, known as the active gravitational mass, takes the form

$$M(r) = 4\pi \int_0^r \left[\rho + \frac{E^2}{8\pi} \right] r^2 dr \quad (14)$$

Let us now consider the problem of charged fluid sphere under conformal motion through CKV which can be given by

$$L_{\xi} g_{ij} = \xi_{i,j} + \xi_{j,i} = \psi g_{ij}. \quad (15)$$

The above eq. (15) give the following set of expressions

$$\xi^1 \nu' = \psi,$$

$$\xi^4 = C_1 = \text{constant},$$

$$\xi^1 = \frac{\psi r}{2},$$

$$\xi^1 \lambda' + 2\xi_1^1 = \psi.$$

These, therefore, readily imply that

$$\theta^v = C_2^2 r^2, \quad (16)$$

$$\theta^\lambda = \frac{C_3^2}{\psi^2}, \quad (17)$$

$$\xi^i = C_1 \delta_4^i + \left(\frac{\psi r}{2} \right) \delta_1^i \quad (18)$$

where C_2 and C_3 are integration constants.

3. Electromagnetic mass models with conformal motions

Now using solutions (16) and (17), the eqs. (28)–(9) take the following form

$$\frac{1}{r^2} \left[1 - \frac{\psi^2}{C_3^2} \right] - \frac{2\psi\psi'}{r C_3^2} = 8\pi\rho + E^2, \quad (19)$$

$$\frac{1}{r^2} \left[1 - \frac{3\psi^2}{C_3^2} \right] = -8\pi p + E^2, \quad (20)$$

$$\frac{\psi^2}{C_3^2 r^2} + \frac{2\psi\psi'}{r C_3^2} = 8\pi p + E^2. \quad (21)$$

From the above eqs. (19)–(21), in a straight forward way, one can get the values for E , ρ and p as

$$E^2 = \frac{1}{r^2} - \frac{1}{r^2} \left(1 - \frac{2\psi^2}{C_3^2} \right) - \frac{2\psi\psi'}{r C_3^2} \quad (22)$$

$$8\pi\rho = \frac{1}{2r^2} - \frac{3\psi\psi'}{r C_3^2} \quad (23)$$

$$8\pi p = -\frac{1}{2r^2} + \frac{\psi\psi'}{r C_3^2} + \frac{2\psi^2}{r^2 C_3^2} \quad (24)$$

In the following subsections we shall consider three different cases to get exact analytical solutions in connection to relativistic charged fluid spheres with conformal motion

3.1 Case-I $\rho = -p$

It is already mentioned in the introductory part that there is a special kind of solution known in the literature as *electromagnetic mass* (EMM) models where all the physical parameters, including the gravitational mass, are arising from the electromagnetic field alone have been extensively studied by several researchers [11,14–23]. In this connection it is interesting to note that most of these investigators exploit an equation of state $p = -\rho$ with the equation of state parameter $\omega = -1$ which is a very common feature in the context of accelerating phase of the present Universe². The equation of state of this type implies that the matter distribution under consideration is in tension and hence the matter is known in the literature as a 'false vacuum' or 'degenerate vacuum' or ' ρ -vacuum' [36–39].

To consider the above mentioned equation of state let us use the eqs (19) and (20) which provide the unique relation

$$\frac{2\psi}{r^2 C_4^2} (\psi - r\psi') = 8\pi r (\rho + p) \quad (25)$$

Therefore, the eq (25), due to the *ansatz* $\rho + p = 0$, gives the value for ψ as either $\psi = 0$ or $\psi = \psi_0 r$, where ψ_0 is an integration constant.

Now, by the use of non-zero value of ψ the exact analytical form for all the physical parameters can be given as

$$8\pi\rho = \frac{1}{2r^2} - 3C_4^2, \quad (26)$$

$$8\pi p = -\frac{1}{2r^2} + 3C_4^2, \quad (27)$$

$$E^2 = \frac{q^2}{r^2} = \frac{1}{2r^2} \quad (28)$$

$$\sigma = \frac{C_4}{4\pi\sqrt{2}r}, \quad (29)$$

² In general, the matter density $\rho > 0$ and fluid pressure $p < 0$. However, there are some special cases available in the literature where $\rho < 0$ and hence $p > 0$ [32–35].

$$e^v = e^{-\lambda} = C_4^2 r^2, \quad (30)$$

where $\psi_0/C_3 = C_4$.

Hence, the total gravitational mass $m(r=a)$, which we get after matching of the interior solution to the exterior Reissner-Nordström solution on the boundary, can be calculated as

$$m(a) = M(a) + \frac{q(a)^2}{2a} = \frac{1}{2\sqrt{2}}(3 - 4C_4^2 q^2) q. \quad (31)$$

It can be observed from the eq. (29) that vanishing charge density σ makes $\psi_0 = 0 = C_4$. This implies $E^2 = 8\pi\rho = -8\pi p = 1/2r^2$ and $e^v = e^{-\lambda} = 0$. Thus ρ, p and E^2 have $1/r^2$ behaviour with proportionality constants $1/16\pi, -1/16\pi$ and $1/2$, respectively. In case of non-zero C_4 the values of $8\pi\rho$ and $8\pi p$ would be shifted in opposite directions by an amount $3C_4^2$ with respect to E^2 . We also observe that σ has got $1/r$ behaviour and $e^v = e^{-\lambda} \propto r^2$. Interestingly, the vanishing charge q does turn the total gravitational mass m , as expressed in the above eq. (31), into an EMM as expected under the ansatz $\rho + p = 0$. In a similar way all the other physical parameters involved in the eqs. (26)–(30) vanish due to vanishing charge. These results imply that conformal motion does admit this type of EMM model. However, a close observation demands that as mass must not be negative so the above expression for mass puts a limit on C_4 with a fixed charge q , as the second term must be smaller than the first one, which implies $C_4 < \sqrt{3}/2q$. Thus charge q limits the proportionality constants for σ, e^v and $e^{-\lambda}$ and also the departure of $8\pi\rho$ and $8\pi p$ from E^2 . We plot all these observable parameters in Figure 1.

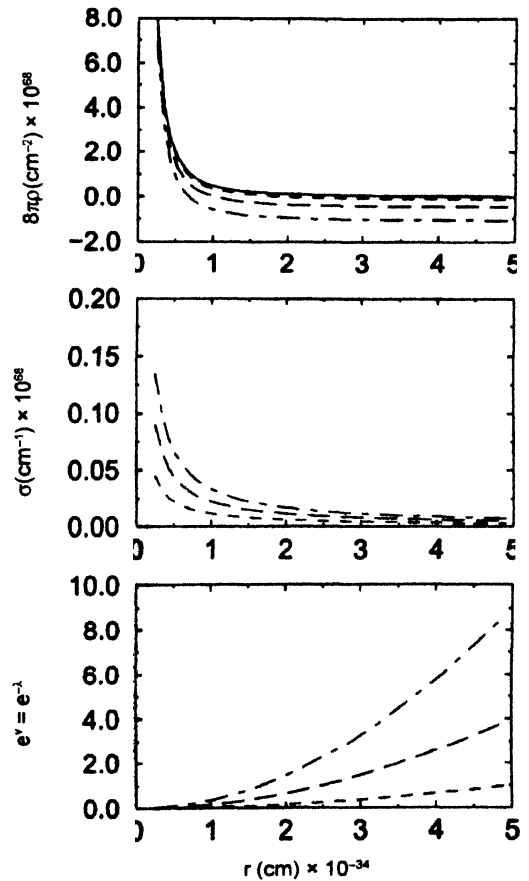


Figure 1. The upper, middle and lower panels represent plots for $8\pi\rho (= -8\pi p)$, σ and $e^v (= e^{-\lambda})$. The solid, dashed, long-dashed and chain curves correspond to $C_4 = 0.0, 0.2, 0.4$ and 0.6 cm^{-1} , respectively.

3.2. Case-II : $\sigma e^{\lambda/2} = \sigma_0$:

Let us consider here the *ansatz* in such a way that the charge density remains constant, say σ_0 [17,35,23]. This is the charge density of the spherical fluid distribution at the centre $r = 0$.

For the above assumption of constant charge density case the eq. (11) leads to a proportionality between E^2 and r^2 as given by

$$E^2 = Ar^2, \quad (32)$$

where $A = 16\pi^2\sigma_0^2/9$ is the proportionality constant.

Substitution of eq. (22) in eq. (32) yields

$$2Ar^2 = \frac{1}{r^2} \left(1 - \frac{2\psi^2}{C_3^2} \right) + \frac{2\psi\psi'}{r C_3^2} \quad (33)$$

By solving above eq. (33) we get

$$\psi^2 = Cr^2 + \frac{C_3^2}{2} [1 + 2Ar^4] \quad (34)$$

where C is an integration constant.

Expressions for ρ and p in eqs. (23) and (24), thus take the following forms

$$8\pi\rho = \frac{1}{2r^2} - 2Ar^2 - 3C_5, \quad (35)$$

$$8\pi p = \frac{1}{2r^2} + 4Ar^2 + 3C_5 \quad (36)$$

where $C_5 = C/C_3^2$.

Therefore, the total gravitational mass $m(r = a)$ here can be given by

$$m(a) = \frac{a}{4} - \frac{C_5 a^3}{2} + \frac{12A}{5} a^5. \quad (37)$$

The condition, $m(a) > 0$ restricts C_5 as

$$C_5 < \frac{1}{2a^2} + \frac{24A}{5} a^2. \quad (38)$$

For the typical values of the physical parameters $q = 1.38 \times 10^{-34}$ cm, $a = 1.0 \times 10^{-16}$ cm and $\sigma_0 = 3q/4\pi a^3 = 3.29 \times 10^{13}$ cm $^{-2}$ (in relativistic units) we get $A = 1.9 \times 10^{28}$ cm $^{-2}$ and hence $C_5 < 5.0 \times 10^{31}$. The eq. (35) for $8\pi\rho$ and eq. (36) for $8\pi p$ are represented by the interference of three terms. The first and second term become comparable to each other around $r = r_{critical}$. For $r \ll r_{critical}$, second term in both the eqs. (35) and (36) are negligibly small compared to the first term i.e. $1/r^2$. In this case, eq. (35) reads as $8\pi\rho = 1/r^2 - 3C_5$, which is the same expression obtained for Case-I ($\rho = -\rho$) but with a very small limit on the integration constant. On the other hand, eq. (36) reads as $8\pi p = 1/r^2 + 3C_5$ but with a different sign for the integration constant. Here, unlike Case-I, we do not have the condition $\rho = -p$. For the case, $r \gg r_{critical}$ it is the second term that dominates and the first term becomes negligibly small.

Now, the relationship between the electric charge and intensity of electric field being $q(r)^2 = E^2 r^2$, the eq. (32) reduces for the total charge to

$$q(a) = \sqrt{A} a^3. \quad (39)$$

Here $q = 0$ does not imply $a = 0$ as we obtained it in the previous case ($q = a/\sqrt{2}$ in the eq (28)) rather it provides the result $\sigma_0 = 0$. Therefore, substitution of this $a \approx \left(\left[1/\sqrt{A} \right]^{1/3} = [3q/4\pi\sigma_0]^{1/3} \right)$ in the eq. (37) does not make the above gravitational mass to vanish for the vanishing electric charge. However, for $\sigma_0 = 0$ the constant A becomes zero. This immediately implies that for the gravitational mass to vanish the condition to be imposed here is $C_5 = 1/2a^2$ whereas for the energy density and fluid pressure this is $C_5 = \pm 1/6r^2$. Therefore, we get a unique condition for EMM model which can be given by $r_{critical} = a/\sqrt{3}$ and can be referred as the *critical radius* for EMM in connection to Lorentz's type extended electron. This means that not the mass of the whole spherical body rather only a sphere of radius up to $0.57a$ (with $a \sim 10^{-16}$ cm [40]) is of electromagnetic in origin and the rest of the mass in the form of the shell is of ordinary gravitational mass.

3.3. Case-III : $8\pi p - E^2 = p_0$:

Following the previous case of constant charge density let us consider here the *ansatz* for a constant pressure, say p_0 which is the central pressure of the fluid sphere and makes balance between the attractive fluid pressure and repulsive Coulombian force as an effective pressure ($p_{effective}$). In reality the physical situation may not be so simple rather very complicated one. However, we assume $8\pi p - E^2 = p_0$ for mathematical simplicity and would like to study an idealized charged fluid model with conformal motion.

Now, from eq. (17), one can get an expression for ψ as

$$\psi^2 = \frac{C_3^2}{3} (1 + \rho_0 r^2). \quad (40)$$

Hence all the other parameters can be obtained as

$$8\pi\rho = -\rho_0 + \frac{1}{2r^2}, \quad (41)$$

$$8\pi p = \rho_0 + \frac{1}{6r^2}, \quad (42)$$

$$\theta'' = C_2^2 r^2, \quad (43)$$

$$e^{-\lambda} = \frac{1}{3} (1 + \rho_0 r^2), \quad (44)$$

$$E^2 = \frac{1}{6r^2}, \quad (45)$$

$$\sigma = \frac{C_4}{4\pi\sqrt{6}r}. \quad (46)$$

The total gravitational mass $m(r = a)$ in this case can be expressed as

$$m(a) = \frac{1}{6} \left(\frac{5a}{2} - \rho_0 a^3 \right). \quad (47)$$

We observe that E^2 and σ with respect to their values of the case I (i.e. $\rho = -\rho$) are $1/3$ and $1/\sqrt{3}$ times smaller, respectively. The $\theta'' \propto r^2$ with a proportionality constant C_2^2 . For $E^2 = 8\pi\rho$, ρ_0 is zero. This implies that $E^2 = 8\pi\rho = 24\pi p$ too has $1/r^2$ behaviour and $e^{-\lambda}$ with a value $1/3$ is a constant. For the case $E^2 \neq 8\pi\rho$, eq. (47) applies an upper limit on the choice of ρ_0 such that $\rho_0 = 8\pi\rho - E^2 < 5/2a^2$. The lower limit being 0 (as negative ρ_0 means $8\pi\rho < E^2$) we should limit ρ_0 between 0 and $5/2a^2$. It can, therefore, be easily observed that for the condition $\rho_0 = 5/2a^2$ the above gravitational mass becomes EMM whereas for ρ the condition is $\rho_0 = 3E^2$ and that for p is $\rho_0 = -E^2$. We plot all the observable of this case in Figure 2.

4. Concluding remarks

We have analyzed the behaviour of a Lorentz's 'extended electron' within the framework of general theory of relativity and observed that conformal motions do admit historically important 'electromagnetic mass' models. To do so we have studied three special cases, viz, (1) $\rho = -\rho$, (2) $\sigma e^{\lambda/2} = \sigma_0$ and (3) $8\pi\rho - E^2 = \rho_0$. The results of the present investigations are, in a nutshell, as follows

Case-I It has been observed that vanishing charge density σ makes $\psi_0 = 0 = C_4$. This immediately implies $E^2 = 8\pi\rho = -8\pi\rho = 1/2r^2$ and $e' = e^{-\lambda} = 0$. Thus, all the physical parameters, including the total gravitational mass m , vanish due to vanishing charge. These results imply that EMM model admitting conformal motion under the constraint $C_4 < \sqrt{3}/2q$.

Case-II Here the expression for the total charge being $q(a) = \sqrt{A}a^3$ we observe that for $\sigma_0 = 0$ the constant $A (= 16\pi^2\sigma_0^2/9)$ becomes zero. This readily implies that for the gravitational mass to vanish the condition to be imposed here is $C_5 = 1/2a^2$ whereas for the energy density and fluid pressure this is $C_5 = \pm 1/6r^2$. Hence, we get a unique condition for EMM model which

can be expressed as $r_{critical} = a/\sqrt{3}$. This suggests that not the mass of the entire charged sphere rather only a radius up to $0.57a$ is of electromagnetic in origin and the rest of the mass in the form of the shell is of ordinary gravitational mass. In this connection we would like to mention that in the framework of general theory of relativity the electron-like spherically symmetric distribution of matter must contain some negative mass density [32–35]. Bonnor and Cooperstock [33] argue that the negativity of the gravitational mass and hence negative energy density ($\rho < 0$) for electron of radius $a \sim 10^{-16}$ is consistent with the Reissner-Nordström repulsion.

Case-III We observe that for $E^2 = 8\pi\rho$, the constant pressure term ρ_0 is zero. For the case $E^2 \neq 8\pi\rho$ we have an upper limit on the choice of ρ_0 such that $\rho_0 = 8\pi\rho - E^2$

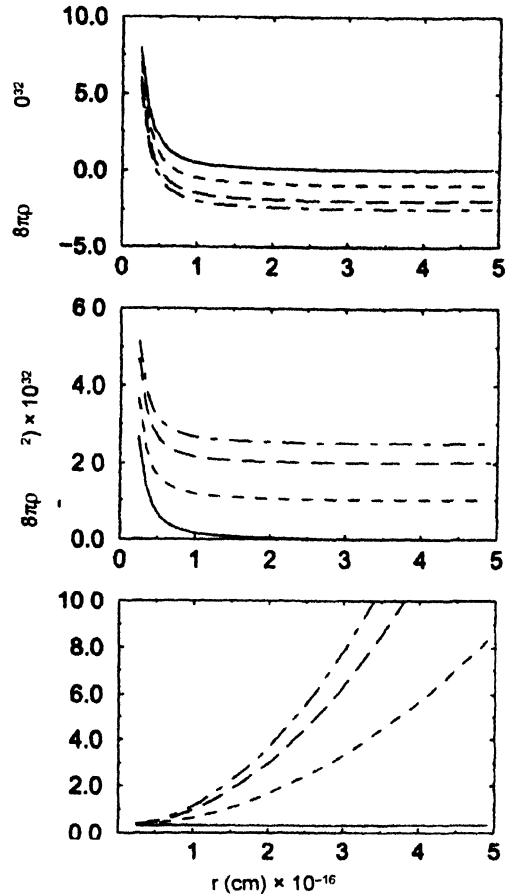


Figure 2 The upper middle and lower panels represent plots for $8\pi\rho$, $8\pi p$ and e . The solid, dashed, long-dashed and chain curves correspond to $\rho_0 = 0.0, 1.0, 2.0$ and 2.5 cm^{-2} respectively.

$< 5/2a^2$. The lower limit being 0 we should limit p_0 between 0 and $5/2a^2$. It can then be observed that for the condition $p_0 = 5/2a^2$ the gravitational mass is of electromagnetic in origin whereas for ρ the condition is $p_0 = 3E^2$ and that for p is $p_0 = -E^2$.

Besides the above discussions we would also like to make here the following comments which appear very much relevant in connection to the present investigations :

(1) The equation of state in the form $p + \rho = 0$ is discussed by Gliner [41] in his study of the algebraic properties of the energy-momentum tensor of ordinary matter through the metric tensors and called it the ρ -vacuum state of matter. It is also to be noted that the gravitational effect of the zero-point energies of particles and electromagnetic fields are real and measurable, as in the Casimir Effect [42]. According to Peebles and Ratra [43], like all energies, this zero-point energy has to contribute to the source term in Einstein's gravitational field equation.

(2) In the Introduction we have mentioned that it is appropriate to use inheritance symmetry to search for the natural relation between geometry and matter. As a well known inheritance symmetry we, therefore, exploit here the symmetry under conformal killing vectors. The features with non-null CKV, as obtained in the present work, show that electron has some probable inheritance symmetry.

(3) In the present investigation we have employed the conformal motion technique in connection to electron-like micro-particle under the 4-dimensional Einstein-Maxwell space-time. We, therefore, feel that it may also be possible to extrapolate the present investigation to the astrophysical bodies, specially quark stars, admitting a one-parameter group of conformal motions. On the other hand, the present investigation with 4-dimensional space-time can be extended to the higher dimensions to see the features with conformal motions.

(4) It is argued by Grøn [44,45] that the negative mass and the associated gravitational repulsion is due to the strain of the vacuum because of vacuum polarization. He also argues that if a vacuum has a vanishing energy, then its gravitational mass will be negative and the observed expansion of the Universe may be explained as a result of repulsive gravitation. It may also be pointed out that according to Ipser and Sikivie [46] domain walls are sources of repulsive gravitation and a spherical domain wall will collapse. To overcome this situation the charged "bubbles" with negative mass keep the wall static and hence in equilibrium.

Acknowledgments

Authors (SR and AAU) are thankful to the authority of Inter-University Centre for Astronomy and Astrophysics, Pune, India for providing them Associateship programme under which a part of this work was carried out.

References

- [1] J J Thomson *Phil. Mag.* 11 229 (1881)
- [2] O Heaviside *Phil. Mag.* 27 324 (1889)

- [3] G F C Searle *Phil Mag* **44** 329 (1897)
- [4] W Kaufmann *Phys Z* **3** 9 (1901a)
- [5] W Kaufmann *Phys Z* **2** 602 (1901b)
- [6] W Kaufmann *Gottinger Nachr* **143** (1901c)
- [7] W Kaufmann *Gottinger Nachr* **291** (1902a)
- [8] W Kaufmann *Phys Z* **4** 54 (1902b)
- [9] M Abraham *Dynamik dee Elektrons* (Gottinger Nachr, 20-41) (1902)
- [10] H A Lorentz *Proc Acad Sci, Amsterdam* **6** (1904) (Reprinted in *The Principle of Relativity* (N Y Dover, INC) p24 (1952)
- [11] A Einstein *Sitz Preuss Akad Wiss* **349** (Reprinted in *The Principle of Relativity* [(N Y Dover, INC) (1952) p190-198] (1919)
- [12] J A Wheeler *Geometrodynamics* (New York Academic) p25 (1962)
- [13] F Wilczek *Phys Today* **52** 11 (1999)
- [14] R P Feynman, R R Leighton and M Sands *The Feynman Lectures on Physics* (Palo Alto Addison-Wesley) Vol. II Chap 28 (1964)
- [15] P S Flondes *Proc Camb Phil Soc* **58** 102 (1962)
- [16] F I Cooperstock and V de la Cruz *Gen Rel Grav* **9** 835 (1978)
- [17] R N Tiwari, J R Rao and R R Kanakamedala *Phys Rev D* **30** 489 (1984)
- [18] R Gautreau *Phys Rev D* **31** 1860 (1985)
- [19] ϕ Grøn *Phys Rev D* **31** 2129 (1985)
- [20] J Ponce de Leon *J Math Phys* **28** 410 (1987)
- [21] R N Tiwari and S Ray *Astrophys Space Sci* **182** 105 (1991)
- [22] S Ray and B Das *Mon Not R Astron Soc* **349** 1331 (2004)
- [23] S Ray *Int J Mod Phys D* **15** 917 (2006)
- [24] C G Böhmer, T Harko and F S N Lobo *Phys Rev D* **76** 084014 (2007)
- [25] C G Böhmer, T Harko and F S N Lobo [arXiv gr-qc/0711 2424] (2008)
- [26] R Maartens and C M Mellin *Clas Quan Grav* **13** 1571 (1996)
- [27] M Mars and J M M Senovilla [arXiv gr-qc/0201066] (2002)
- [28] M K Mak and T Harko *Int J Mod Phys D* **13** 149 (2004)
- [29] T Harko and M K Mak *Phys Rev D* **69** 064020 (2004)
- [30] M K Mak and T Harko *Phys Rev D* **70** 024010 (2004)
- [31] R Neves [arXiv hep-th/0601040] (2006)
- [32] F I Cooperstock and N Rosen *Int J Theor Phys* **28** 423 (1989)
- [33] W B Bonnor and F I Cooperstock *Phys Lett A* **139** 442 (1989)
- [34] L Herrera and V Varela *Phys Lett A* **189** 11 (1994)
- [35] S Ray and S Bhadra *Phys Lett A* **322** 150 (2004)
- [36] C W Davies *Phys Rev D* **30** 737 (1984)
- [37] J J Blome and W Priester *Naturwissenschaften* **71** 528 (1984)
- [38] C Hogan *Nature* **310** 365 (1984)
- [39] N Kaiser and A Stebbins *Nature* **310** 391 (1984)

- [40] C Quigg *Gauge Theories of the Strong, Weak and Electromagnetic Interactions* (New York Benjamin) p3 (1983)
- [41] E B Gliner *Sov. Phys. JETP* **22** 378 (1966)
- [42] H B G Casimir *Proc. K. Ned. Acad. Wet.* **51** 635 (1948)
- [43] P J E Peebles and B Ratra *Rev. Mod. Phys.* **75** 559 (2003)
- [44] ϕ Grøn *Am. J. Phys.* **54** 46 (1986a)
- [45] ϕ Grøn *Gen. Rel. Grav.* **18** 591 (1986b)
- [46] J Ipser and P Sikivie *Phys. Rev. D* **30** 712 (1984)